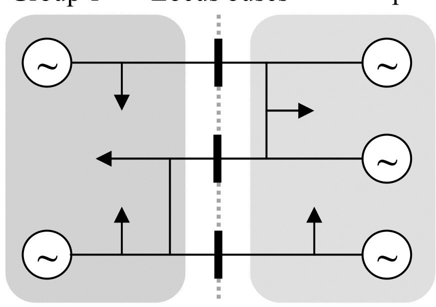
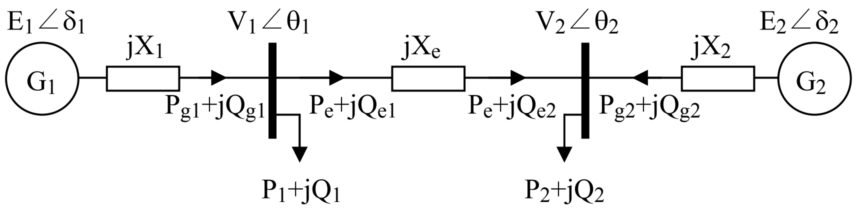
Appendixes

A-1 The Reason Why Infinite Bus Can Be Assumed

In most power system analyses, smaller part is represented by 1 machine and 1 load, and larger part is represented as infinite bus. Larger part never has infinite size. While, how large part can be regarded as infinite bus? Since no references were found, the author ties and introduces. Oscillatory instability, which needs most complex analysis, is taken as example. Oscillatory instability appears as power swing between two generator groups in a large interconnection. Therefore, the simplest model of the phenomenon is made of 2 machines and 2 loads as shown in A-Fig. 1.1. So as to manual calculation is available, generators are represented as voltage source behind transient reactance.



A-Fig. 1.1 Structure of inter area oscillation



A-Fig. 1.2 Analysis model for inter area oscillation

Model for inter area oscillation is shown in A-Fig. 1.1, which is conducted by procedure as follows.

1. Perform modal analysis or time domain simulation on detailed system.
2. Divide generators into two groups having opposite swing vectors.
3. Select locus buses on power swing and join those buses.
4. Make two aggregated system seen from jointed locus buses.

In A-Fig. 1.2, values of active and reactive power are expressed by node voltages and its phase angles. (for i = 1, 2)

$$\left.\begin{aligned} P_{gi} &= \frac{E_i\, V_i \sin(\delta_i - \theta_i)}{X_i}, \quad Q_{gi} = \frac{E_i\, V_i \cos(\delta_i - \theta_i) - V_{i2}}{X_i} \\ P_e &= \frac{V_1 V_2 \sin(\theta_1 - \theta_2)}{X_e}, \quad Q_{e1} = \frac{V_1^2 - V_1 V_2 \cos(\theta_1 - \theta_2)}{X_e}, \quad Q_{e2} = \frac{V_1 V_2 \cos(\theta_2 - \theta_1) - V_2^2}{X_e} \end{aligned}\right\} \quad \text{(A-1.1)}$$

Voltage sensitivities of load's active and reactive power are assumed as follows.

$$P_i = P_{i0}\,V_i^{\alpha}, \quad Q_i = Q_{i0}\,V_i^{\beta} \qquad \text{(for i=1,2)} \qquad \text{(A-1.2)}$$

Small deviations around operating point are taken as variables. Equation (A-1.3) is conducted from active and reactive power balance.

$$\left.\begin{aligned} \Delta P_{g1} - \Delta P_1 - \Delta P_e = 0, \quad \Delta Q_{g1} - \Delta Q_1 - \Delta Q_{e1} = 0 \\ \Delta P_{g2} - \Delta P_2 + \Delta P_e = 0, \quad \Delta Q_{g2} - \Delta Q_2 + \Delta Q_{e2} = 0 \end{aligned}\right\} \quad \text{(A-1.3)}$$

Substituting (A-1.1) and (A-1.2) into (A-1.3), $\Delta\theta_i$ and ΔV_i are expresses as functions of $\Delta\delta_i$ and ΔE_i as equation (A-1.4).

$$\begin{pmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta V_1/V_1 \\ \Delta V_2/V_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \begin{pmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta E_1/E_1 \\ \Delta E_2/E_2 \end{pmatrix} \qquad \text{(A-1.4)}$$

Using (A-1.1), (A-1.2), and (A-1.3), ΔP_i and ΔV_i are expresses as functions of $\Delta\delta_1 - \Delta\delta_2$ as equation (A-1.5). Here, it must be noted that $B_{i1} + B_{i2} = 0$, therefore, ΔP_i and ΔV_i are expressed as functions of $\Delta\delta_1 - \Delta\delta_2$.

$$\begin{pmatrix} \Delta P_{g1} \\ \Delta P_{g2} \\ \Delta V_1/V_1 \\ \Delta V_2/V_2 \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix} \begin{pmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta E_1/E_1 \\ \Delta E_2/E_2 \end{pmatrix} \qquad \text{(A-1.5)}$$

Sum of generator capacity of group i is P_{0i} . Unit inertia constant of each generator is assumed as equal. Thus, swing equations of the two groups are expressed as follows.

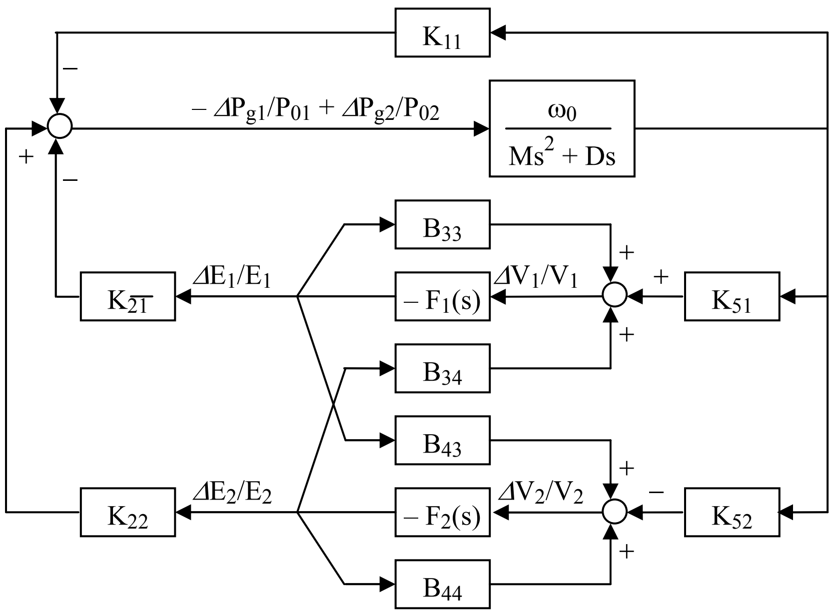
$$(Ms^2 + Ds)\,\frac{\Delta\delta_1}{\omega_0} = -\,\frac{\Delta P_{g1}}{P_{01}}, \quad (Ms^2 + Ds)\,\frac{\Delta\delta_2}{\omega_0} = -\,\frac{\Delta P_{g2}}{P_{02}} \qquad \text{(A-1.6)}$$

Making difference of the two equations in (A-1.6), united equation is conducted as follows.

$$(Ms^2 + Ds)\,\frac{\Delta\delta_1 - \Delta\delta_2}{w_0} = -\,\frac{\Delta P_{g1}}{P_{01}} + \frac{\Delta P_{g2}}{P_{02}} \qquad \text{(A-1.7)}$$

From (A-1.4), (A-1.5), and (A-1.7), A-Fig. 1.3 is conducted. Here, $-F_i(s)$ means average excitation system gain of generators in group i . Coefficients K_{ij} are calculated as follows.

$$\left.\begin{aligned} K_{21} = \frac{B_{13}}{P_{01}} - \frac{B_{23}}{P_{02}}, \quad K_{51} = B_{31}, \quad K_{11} = \frac{B_{11}}{P_{01}} - \frac{B_{21}}{P_{02}} \\ K_{22} = \frac{B_{14}}{P_{01}} - \frac{B_{24}}{P_{02}}, \quad K_{52} = B_{41} \end{aligned}\right\} \quad \text{(A-1.8)}$$



A-Fig. 1.3　Block diagram for calculating damping torque

By A-Fig. 1.3, damping torque is calculated as follows. Since K_{11} is a real number, it never contribute damping. ΔE_1 and ΔE_2 are calculated as follows.

$$\begin{bmatrix} 1/F_1(s) + B_{23} & B_{34} \\ B_{43} & 1/F_2(s) + B_{44} \end{bmatrix} \begin{bmatrix} \Delta E_1/E_1 \\ \Delta E_2/E_2 \end{bmatrix} = \begin{bmatrix} -K_{51} \\ K_{52} \end{bmatrix} (\Delta\delta_1 - \Delta\delta_2) \qquad \text{(A-1.9)}$$

Therefore, synchronizing torque S_i and damping torque D_i through excitation system are calculated as follows (for i = 1, 2). Here, C is the matrix in left side of equation (A-1.9).

$$\begin{bmatrix} S_1 + j\,D_1 \\ S_2 + j\,D_2 \end{bmatrix} = \begin{bmatrix} K_{21} & 0 \\ 0 & -K_{22} \end{bmatrix} C^{-1} \begin{bmatrix} -K_{51} \\ K_{52} \end{bmatrix} (\Delta\delta_1 - \Delta\delta_2) \qquad \text{(A-1.10)}$$

Total damping torque of both groups D_{12} is sum of D_1 and D_2 shown as follows.

$$D_{12} = D_1 + D_2 \qquad \text{(A-1.11)}$$

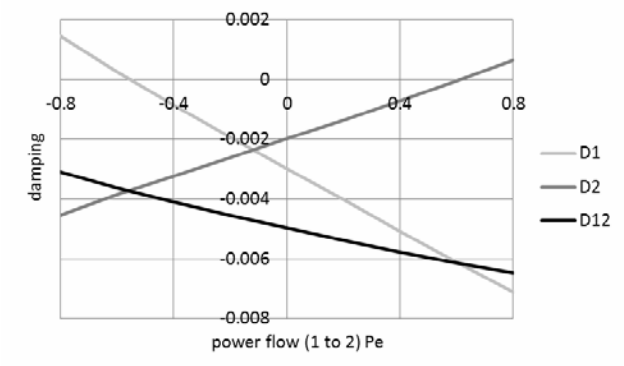
[Generator Capacity Ratio of the two groups is 2:3]　　Damping by tie line flow is shown in A-Fig. 1.4. Tie line flow is adjusted by varying load amount. Constants are shown as follows.

$X_1 = 0.225$,　$Xe = 1.0$,　$X_2 = 0.15$,　$E_1 = V_1 = V_2 = E_2 = 1.0$,

$P_{g1} = 2$,　$P_{g2} = 3$,　$P_{01} = 2$,　$P_{02} = 3$,　$\alpha = 1$,　$\beta = 2$,　$P_1 + P_2 = 5$,

AVR : $G = 30$,　$T = 0.3$ 秒 ,　swing speed : $\omega_s = 2\pi * 0.5$Hz

A-Fig. 1.4 shows a slight asymmetry due to capacity asymmetry. As the result, total damping D_{12} takes large negative value as much power flows from group 1 (smaller) to group 2 (larger). Although capacity asymmetry is so light as 2:3, smaller group 1 shows ill damping (D_1), and spoils oscillatory stability of the interconnection very much.



A-Fig. 1.4 Damping by tie-line flow (Cap. 2:3)

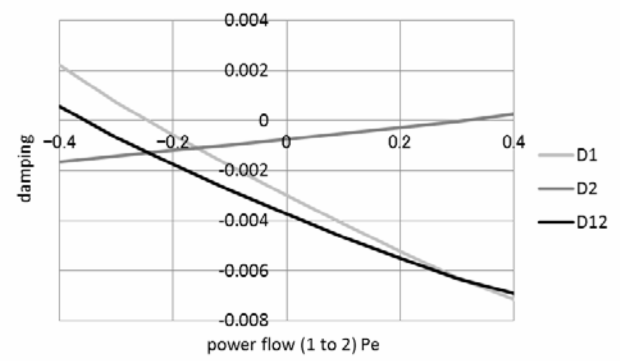
[Generator Capacity Ratio of the two groups is 1:4] Damping by tie line flow is shown in A-Fig. 1.5. Tie line flow is adjusted by varying load amount. Constants are shown as follows.

$X_1 = 0.45$, $X_e = 2.0$, $X_2 = 0.1125$, $E_1 = V_1 = V_2 = E_2 = 1.0$,

$P_{g1} = 1$, $P_{g2} = 4$, $P_{01} = 1$, $P_{02} = 4$, $\alpha = 1$, $\beta = 2$, $P_1 + P_2 = 5$,

AVR : G = 30 , T = 0.3 秒 , swing speed : $\omega_s = 2\pi * 0.5$Hz

A-Fig. 1.5 shows more remarkable asymmetry than A-Fig. 1.4 due to remarkable capacity asymmetry. Group 2 damping (D_2) hardly changes by tie-line flow, and as the result, total damping D_{12} is almost equal to group 1 damping (D1). Oscillatory stability of the total interconnection is decided by only smaller group 1, and as the result, larger group 2 can be regarded as infinite bus. Therefore, 1 machine, 1 load, and infinite bus model is evaluated as an adequate model.



A-Fig. 1.5 Damping by tie-line flow (Cap. 1.4)

A-2 Fundamental Equations of Synchronous Machine

In case of transient stability, generator can be simply regarded as voltage source behind transient reactance. However in case of oscillatory stability, field winding must be modeled. Damper windings can be considered as damping D. Park model that is most commonly adopted considers damper windings, but ignores magnetic flux linking any two of armature, field, and damper windings. The ignorance is not verified yet. Synchronous machine model considering field winding and ignoring damper windings is minutely introduced by Kimbark, but in the last step, careful explanation conducting “Fundamental Equation” is omitted. The appendix will fulfill the flaw.

[Direct Axis and Quadrature Axis] Synchronous machine has magnetic pole, which induction machine does not have, and character is much different on pole direction (direct axis) and its cross direction (quadrature axis). Quadrature axis is not affected by field, and its constants are different from direct axis constants, especially in salient pole machine. The two axes must be modeled independently.

Voltage, current, and magnetic flux linkage are defined as vector as follows.

$$\mathbf{V_t} = V_d + j\,V_q \qquad \text{(A-2.1)}$$

$$\mathbf{I} = I_d + j\,I_q \qquad \text{(A-2.2)}$$

$$\mathbf{\Psi} = \Psi_d + j\,\Psi_q \qquad \text{(A-2.3)}$$

If our viewpoint is fixed on armature (stator), since magnetic pole is rotating, inductance in each a, b, c phase becomes functions of pole position θ. Although we want to knowledge of electric circuit, such inductance varying by θ brings a considerable difficulty in calculation. On the contrary, if our viewpoint is fixed on magnetic pole (rotor), direct and quadrature axis reactance becomes constant no matter what pole position θ is. The coordinates-axis transform is called as “Park’s equation” by giving him credit for being the pioneer.

[Transformer- and Speed- Electromotive Force] According to Faraday’s electromagnetic induction law, time variation of magnetic flux linking coil induces voltage in the coil. Another word, time variation of current I in coil with inductance L induces electromotive force V in the coil. Since magnetic flux linkage is Ψ = L I, the electromotive force is expressed as follows.

$$V = L\frac{d\,I}{d\,t} \qquad \text{or} \qquad V = \frac{d\,\Psi}{d\,t} \qquad \text{(A-2.3)}$$

However, it is careless to regard time differential of (A-2.3) as electromotive force. The fact that coordinates-axis is rotating is forgotten. The rotation can be considered by multiplying $e^{j\theta}$ as rotation. Thus, relation of flux linkage and voltage is expressed as follows. Laplace’s operator s is used as time differential hereafter.

$$(V_d + jV_q)\,e^{j\theta} = s\{(\Psi_d + j\Psi_q)\,e^{j\theta}\} = s\,(\Psi_d + j\Psi_q)\,e^{j\theta} + j(\Psi_d + j\Psi_q)\,e^{j\theta}\,s\theta$$

$$\therefore \quad V_d = s\Psi_d - \Psi_q\,s\theta \qquad \text{(A-2.4)}$$

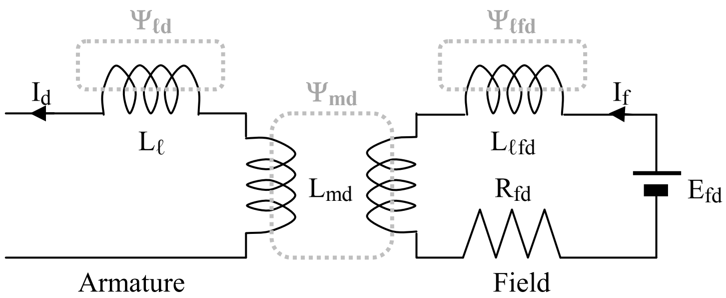
$$V_q = s\Psi_q + \Psi_d\,s\theta \qquad \text{(A-2.5)}$$

The first term in right side of (A-2.4) and (A-2.5) appears when machine is not rotating, and is called as "transformer electromotive force". The second term appears by magnetic pole rotation, and is called as "speed electromotive force". The latter is far larger than the former. Therefore in many cases, electromotive force is simply expressed as follows.

$$V_d = -\Psi_q \, s\theta \qquad \text{(A-2.4')}$$

$$V_q = \Psi_d \, s\theta \qquad \text{(A-2.5')}$$

[Equivalent Circuit] Usually, direct axis of synchronous machine is expressed equivalent circuit as A-Fig. 2.1. Field and armature is magnetically combined and have mutual inductance L_{md} and its flux Ψ_{md}. Some of flux made by field current does not link armature and forms leakage flux $\Psi_{\ell fd}$. Samely some of flux made by direct axis current I_d forms leakage flux $\Psi_{\ell d}$. On the contrary of rotor, since armature (stator) is symmetrically built, leakage inductance is not different in direct and quadrature axis. Since the equivalent circuit is seen from viewpoint on rotating pole, armature flux, voltage, and current are seen as direct current from the viewpoint.



A-Fig. 2.1 Equivalent circuit of direct axis

Flux linkages, which are product of inductance and current, are expressed as follows.

$$\Psi_{md} = L_{md} \, (I_f - I_d)$$

$$\Psi_{fd} = L_{fd} \, I_f$$

$$\Psi_{\ell} = -L_{\ell} \, I_d$$

Minus sign on I_d means that machine is regard as generator, in which power flow direction toward outside is expressed positive.

Armature's direct axis linkage flux Ψ_d is sum of Ψ_{ℓ} and Ψ_{md} , and is expressed as follows.

$$\Psi_d = \Psi_{\ell} + \Psi_{md} = -L_{\ell} \, I_d + L_{md} \, (I_f - I_d)$$

While, direct axis synchronous reactance is expressed as follows.

$$L_d = L_{md} + L_{\ell}$$

Therefore, direct axis flux linkage is expressed as follows.

$$\Psi_d = L_{md} I_f - L_d I_d \tag{A-2.6}$$

Quadrature axis is not affected by field, and is expressed as follows.

$$\Psi_q = - L_q I_q \tag{A-2.7}$$

Flux linking to field winding Ψ_{fd} is sum of $\Psi_{\ell fd}$ and Ψ_{md} , and is expressed as follows.

$$\Psi_{fd} = L_{\ell fd} I_f + L_{md} (I_f - I_d)$$

Open circuit inductance of field circuit is expressed as follows.

$$L_{fd} = L_{\ell fd} + L_{md}$$

Therefore, flux linking to field winding also can be expresses as follows.

$$\Psi_{fd} = L_{fd} I_f - L_{md} I_d \tag{A-2.8}$$

Field voltage E_{fd} is sum of voltage drop through field resistance R_{fd} and time variation of field flux linkage, and is expressed as follows.

$$E_{fd} = R_{fd} I_f + s\Psi_{fd} \tag{A-2.9}$$

[Equations of Current and Flux] Here, per unit method is introduced. As bases, following terms are convenient for calculation.

As terminal voltage,	rated terminal voltage	V_{t0}
As armature current,	rated current	I_0
As rotating speed,	rated rotating speed	ω_0
As field current,	current at no load and rated voltage	I_{f0}
As field voltage,	voltage at no load and rated voltage	E_{fd0}
As armature flux,	direct axis flux at no load and rated voltage	Y_{d0}

At no load, $I_q = 0$, therefore, quadrature axis flux is also zero.
At no load, rated speed, and rated voltage, relations on flux, voltage, and current are expressed as follows.

$$V_{t0}/\omega_0 = \Psi_{d0} = \Psi_{md0} = L_{md} I_{f0} \tag{A-2.10}$$

(Remark) While $\Psi_{d0} = \Psi_{\ell 0} + \Psi_{md0}$, $I_{d0} = 0$ results $\Psi_{\ell 0} = 0$.

$$\Psi_{fd0} = L_{fd} I_{f0} \tag{A-2.11}$$

$$E_{fd0} = R_{fd} I_{f0} = R_{fd} \Psi_{fd0}/L_{fd} \tag{A-2.12}$$

Dividing (A-2.6) by (付 2.10), equation as follows are conducted.

$$\frac{\Psi_d}{\Psi_{d0}} = \frac{L_{md} I_f}{L_{md} I_{f0}} = \frac{L_d I_d}{V_{t0}/\omega_0} = \frac{I_f}{I_{f0}} = \frac{\omega_0 L_d}{V_{t0}/I_0} * \frac{I_d}{I_0} \tag{A-2.13}$$

Here, since Ψ_d/Ψ_{d0} means direct axis flux linkage in per unit ψ_d ,
I_f/I_{f0} means field current in per unit i_f ,
I_d/I_0 means direct axis current in per unit i_d

and $\dfrac{\omega_0 L_d}{V_{t0}/\omega_0}$ means direct axis synchronous reactance x_d

therefore, equation (A-2.13) can also be expressed as follows.

$$\psi_d = i_f - x_d\, i_d \tag{A-2.14}$$

Dividing (A-2.7) by (A-2.10), equation as follows is obtained.

$$\frac{\Psi_q}{\Psi_{d0}} = -\frac{L_q I_q}{V_{t0}/\omega_0} = -\frac{\omega_0 L_q}{V_{t0}/I_0} * \frac{I_q}{I_0} \tag{A-2.15}$$

Here, since Ψ_q/Ψ_{d0} means quadrature axis flux linkage in per unit ψ_q ,
I_q/I_0 means quadrature axis current in per unit i_q ,

and $\dfrac{\omega_0 L_q}{V_{t0}/I_0}$ means quadrature axis synchronous reactance in per unit x_q ,

therefore, equation (A-2.15) can also be expressed as follows.

$$\psi_q = -\, x_q\, i_q \tag{A-2.16}$$

Dividing (A-2.8) by (A-2.10), wquation as follows is obtained.

$$\frac{\Psi_{fd}}{\Psi_{md0}} = \frac{L_{fd} I_f}{L_{md} I_{f0}} - \frac{L_{md} I_d}{V_{t0}/\omega_0} \tag{A-2.17}$$

Here, considering

$$I_{f0} = \frac{\Psi_{md0}}{L_{md}} = \frac{\Psi_{fd0}}{L_{fd}} ,$$

equation (A-2.17) can also be expressed as follows.

$$\frac{\Psi_{fd} L_{fd}}{L_{d0} L_{md}} = \frac{L_{fd} I_f}{L_{md} I_{f0}} - \frac{L_{md} I_d}{V_{t0}/\omega_0}$$

$$\therefore \quad \frac{\Psi_{fd}}{\Psi_{fd0}} = \frac{I_f}{I_{f0}} - \frac{\omega_0 L_{md}^2 I_d}{L_{fd} V_{t0}} \tag{A-2.18}$$

Direct axis transient reactance L_d' can be expressed as follows.

$$L_d' = L_\ell + \frac{L_{md} L_{\ell fd}}{L_{md} + L_{\ell fd}}$$

$$\therefore \quad L_d - L_d' = L_{md} - \frac{L_{md}\, L_{\ell fd}}{L_{md} + L_{\ell fd}} = \frac{L_{md}^{\,2}}{L_{fd}} \qquad \text{(A-2.19)}$$

Substituting (A-2.19) to (A-2.18), equation as follows is obtained.

$$\frac{\Psi f_d}{\Psi_{fd0}} = \frac{I_f}{I_{f0}} - \frac{\omega_0\,(L_d - L_d')}{V_{t0}/I_0} * \frac{I_d}{I_0} \qquad \text{(A-2.20)}$$

Here, since $\frac{\Psi_{fd}}{\Psi f_{d0}}$ means field flux linkage in per unit ψ_{fd} ,

$\frac{I_f}{I_{f0}}$ means field current in per unit i_f ,

$\frac{\omega_0 L_d}{V_{t0}/I_0}$ means direct axis synchronous reactance in per unit x_d ,

and $\frac{\omega_0 L_d'}{V_{t0}/I_0}$ means direct axis transient reactance in per unit x_d'

therefore, equation (A-2.20) can also be expressed as follows.

$$\psi_{fd} = i_f - (x_d - x_d')\, i_d \qquad \text{(A-2.21)}$$

Dividing (A-2.19) by (A-2.12), equation as follows is obtained.

$$\frac{E_{fd}}{E_{fd0}} = \frac{R_{fd}\, I_f}{R_{fd}\, I_{f0}} + \frac{s\, \Psi_{fd}\, L_{fd}}{R_{fd}\, \Psi_{fd0}} \qquad \text{(A-2.22)}$$

Here, using open circuit field time constant

$$T_{do}' = \frac{L_{fd}}{R_{fd}} \quad ,$$

(A-2.22) can also be expressed as follows.

$$\frac{Ef_d}{E_{fd0}} = \frac{R_{fd}\, I_f}{R_{fd}\, I_{f0}} + T_{do}'\, s\, \frac{\Psi_{fd}}{\Psi_{fd0}} \qquad \text{(A-2.23)}$$

Here, since Ef_d/E_{fd0} means field voltage in per unit e_{fd} ,

I_f/I_{f0} means field current in per unit i_{fd} ,

and Ψ_{fd}/Ψ_{fd0} means field flux linkage in per unit ψ_{fd} ,

therefore, equation (A-2.23) can also be expressed as follows.

$$e_{fd} = i_f + T_{do}' \, s \, \psi_{fd} \qquad \text{(A-2.24)}$$

[Equation of Torque] Swing equation expresses “mass * acceleration = force” in rotating system, and is expressed by using rotating speed in per unit n as follows.

$$M \, s \, n = \frac{M \, s^2 \, \delta}{\omega_0} = T_m - T_e \qquad \text{(A-2.25)}$$

Unit inertia constant M means the time while rotating speed rises zero to rated speed with giving 1 per unit acceleration torque, and is equal to 2H in JEC.

Meaning of mechanical input torque will be clear. Meaning of electric output torque will need some additional explanation.

At first, it must be noticed that “power = torque * rotating speed”. Electric active and reactive output of synchronous machine can be expressed as follows ignoring damping torque.

$$P_e + j \, Q_e = \mathbf{v_t} \, \mathbf{I}^* = (v_d + jv_q) \, (i_d - i_q) = (v_d \, i_d + v_q \, i_q) + j \, (v_q \, i_d - v_d \, i_q)$$

$$\therefore \quad P_e = v_d \, i_d + v_q \, iq \qquad \text{(A-2.26)}$$

(A-2.4’) and (A-2.5’) can be expressed in per unit method as follows.

$$v_d = -\psi_q \, n$$
$$v_q = \psi_d \, n$$

“Power = torque * rotating speed” is expressed mathematically as follows.

$$P_e = T_e \, n$$

Substituting these three equations above into (A-2.26), equation as follows is obtained.

$$T_e \, n = -\psi_q \, n \, i_d + \psi_d \, n \, i_q$$

Dividing both sides and adding damping term, equation of torque is obtained as follows.

$$T_e = -\psi_q \, i_d + \psi_d \, i_q + \frac{D \, s \, \delta}{\omega_0} \qquad \text{(A-2.27)}$$

Here exist relations between rotor position θ, generator internal phase angle δ, rotating speed n, speed deviation Δn.

$$\theta = \frac{\omega_0 \, t + \delta}{\omega_0} \qquad \text{(A-2.28)}$$

$$s \, \theta = 1 + \frac{s \, \delta}{\omega_0} = 1 + \Delta n = n \qquad \text{(A-2.29)}$$

[Fundamental Equations of Synchronous Machine]　Thus, all fundamental equations of synchronous machine are obtained. They are summarized as follows.

$$\mathbf{v_t} = v_d + j\, v_q \qquad \text{(A-2.1')}$$

$$\mathbf{i} = i_d + j\, i_q \qquad \text{(A-2.2')}$$

$$\boldsymbol{\psi} = \psi_d + j\, \psi_q \qquad \text{(A-2.3')}$$

$$v_d = s\, \psi_d - \psi_q\, s\, \theta \qquad \text{(A-2.4')}$$

$$v_q = s\, \psi_q + \psi_d\, s\, \theta \qquad \text{(A-2.5')}$$

$$\psi_d = i_f - x_d\, i_d \qquad \text{(A-2.14)}$$

$$\psi_q = \quad - x_q\, i_q \qquad \text{(A-2.16)}$$

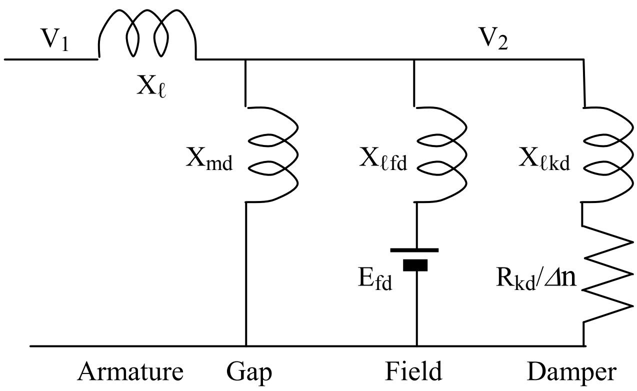
$$\psi_{fd} = i_f - (x_d - x_d')\, i_d \qquad \text{(A-2.21)}$$

$$e_{fd} = i_f + T_{do}'\, s\, \psi_{fd} \qquad \text{(A-2.24)}$$

$$\frac{M\, s\, \delta}{\omega_0} = T_m - T_e \qquad \text{(A-2.25)}$$

$$T_e = \psi_d\, i_q - \psi_q\, i_d + \frac{D\, s\, \delta}{\omega_0} \qquad \text{(A-2.27)}$$

[Damper Windings and Damping Coefficient]　Damping coefficient D is introduced in (A-2.27). Although damping torque varies by operational conditions and is difficult to evaluate, value at no load and no excitation can be calculated using induction motor theory.



A-fig. 2.2　Equivalent circuit of direct axis

Equivalent circuit of direct axis of synchronous machine is shown in A-Fig. 2.2. Relations as follows w exist between reactance, resistance, and synchronous machine constants.

$$X_d = X_\ell + X_{md} \qquad \text{(A-2.28)}$$

$$X_d' = X_\ell - \frac{1}{1/X_{md} + 1/X_{\ell fd}} \tag{A-2.29}$$

$$X_d'' = X_\ell + \frac{1}{1/X_{md} + 1/X_{\ell fd} + 1/X_{\ell kd}} \tag{A-2.30}$$

$$T_d' = \frac{1}{\omega_0 R_{kd}} \left(X_{\ell kd} + \frac{1}{1/X_{md} + 1/X_{\ell fd} + 1/X_\ell} \right) \tag{A-2.31}$$

Here, ω_0 is angular frequency of power system. Since assuming no load and no excitation, $E_{fd} = 0$.

In calculation of induction motor torque, "secondary resistance by slip" is used. Here, decelerating torque is regarded positive, Δn is used as negative slip. If Δn is very small, blanch including $R_{kd} / \Delta n$ can be regarded as open, and secondary voltage V_2 can be calculated as follows.

$$V_2 \fallingdotseq \frac{X_d' - X_\ell}{X_d' + X_e} V_1$$

Induction motor torque is equal to secondary input power, and is expressed as follows.

$$T_k \fallingdotseq \frac{V_2^2}{R_{kd}/\Delta n}$$

Here, assumed as $V_1^2 \fallingdotseq 1$, damping coefficient is calculated as follows.

$$T_k \fallingdotseq \left(V_1 \frac{X_d' - X_\ell}{X_d' + X_e}\right)^2 \frac{1}{R_{kd}} \Delta n$$

$$\therefore \quad D = \frac{T_k}{\Delta n} \fallingdotseq \left(V_1 \frac{X_d' - X_\ell}{X_d' + X_e}\right)^2 \frac{1}{R_{kd}}$$

To make damping coefficient large, conditions as follows are needed.

small	X_ℓ	that is,	high	V_2
large	$X_d' - X_d''$	that is,	small	X_{kd}
large	T_d''	that is,	small	R_{kd}
small	X_e	that is,	tight interconnection	

To understand that small R_{kd} results good damping, "proportional shifting of torque by secondary resistance" will be helpful. Field winding can be ignored in quadrature axis.

Since actual value of damping coefficient D is not equal to that at no load and no excitation, the value must be calculated using minute model including damper windings, and is known as 5 or more by experience. Generator with small damping coefficient prone to cause power swing, and needs some consideration.

[Ex. Quadrature Axis Damping Coefficient of a Thermal Generator] Although thermal generator's rotor is a lumped iron and has no damper winding, eddy current on rotor's surface performs the role of damper windings. However in quadrature axis, not only eddy current but also large loop current like field winding on direct axis appear, and the large loop does not contribute to damping. Considering the large loop current, quadrature axis transient reactance X_q' and transient time constant T_q' appear. Machine constants are shown as follows.

$X_q = 1.884$, $X_q' = 0.6$, $X_q'' = 0.238$, $X_\ell = 0.178$, $T_q'' = 0.02$ 秒, $X_e = 0.3$, $V_t = 0.97$

1. Considering X_q'

$X_{md} = 1.626$, $X_{\ell fq} = 0.56991$, $X_{\ell kq} = 0.06994$, $R_{kq} = 0.02581$, $V_1 = 0.97$

$$D \fallingdotseq \left(0.97 * \frac{0.6-0.178}{0.6+0.3}\right)^2 * \frac{1}{0.02581} = 8.01$$

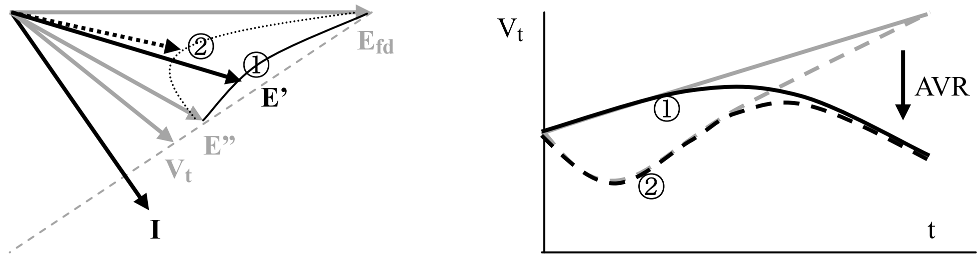
2. Ignoring X_q'

$X_{md} = 1.626$, $X_{\ell kq} = 0.06230$, $R_{kq} = 0.02954$, $V_1 = 0.97$

Since $X_{\ell fd}$ is inifinite (open circuit), $V_2 = V_1$. Then,

$$D \fallingdotseq \left(0.97 * \frac{1.884 - 0.178}{1.884+0.3}\right)^2 * \frac{1}{0.02954} = 19.43$$

Damping is far better by ignoring X_q'. While, which is the truth?



A-Fig. 2.3 Voltage profiles on disconnecting test by considering/ignoring X_q'

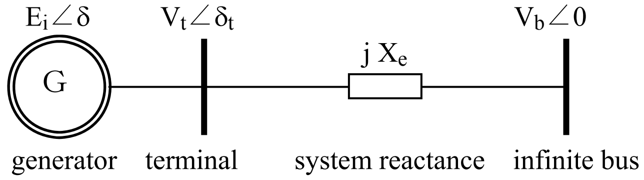
Voltage profile on disconnecting test will be a hint. As shown in A-Fig.2.3, just after disconnection from system, voltage behind subtransient reactance ($X_d'' \fallingdotseq X_q''$): **E**" appears at terminal. As time goes by, terminal voltage shifts to voltage behind transient reactance: **E'** , whose value is calculated as follows.

$$\mathbf{E}' = \mathbf{V}_t + (X_d' I_q + X_q' I_d) + j\,(X_d' I_d - X_q' I_q)$$

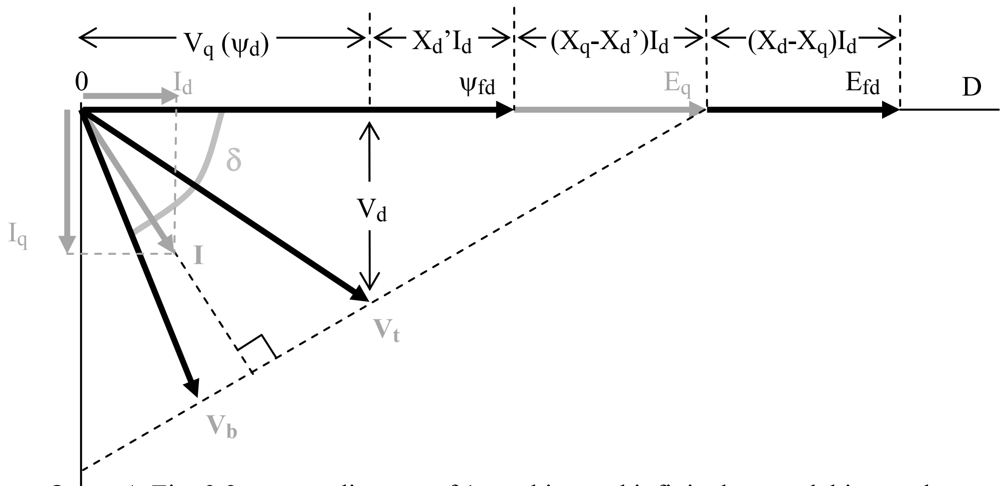
In case of ignoring X_q', X_q' in equation above is displaced by X_q . Although terminal voltage sag must appear in a short time after disconnection by ignoring X_q', such a sag never seen in existing thermal generators. Thus, X_q' surely exists in thermal generator, and considering X_q' results poor damping as example introduced in Fig. 6.1.

A-3 1 Machine and Infinite Bus Model

[Block Diagram of 1 Machine and Infinite Bus Model] Y-connection of trunk system, generator, and load necessarily appears when partial power system is aggregated. Load is included to infinite bus in the model as shown in A-Fig. 3.1. Oscillatory stability is not accurately analyzed by the model. However celebrating historical contribution in modern power system analysis by Heffron - Phillips and de Mello - Concordia, and considering some practical use of the model, the technique is minutely introduced hereafter.



A-Fig. 3.1 1 machine and infinite bus model



Q A-Fig. 3.2 vector diagram of 1 machine and infinite bus model in steady state

Vector diagram of A-Fig. 3.1 is shown as A-Fig. 3.2. Here, an imaginary voltage behind guadrature axis reactance E_q is introduced. Phase angle δ is defined as the angle made by E_q and V_b . Then,

$$E_q = \psi_{fd} + (X_q - X_d') I_d$$

$$\mathbf{I} = I_d + j\, I_q = \frac{\mathbf{E}_q - \mathbf{V}_b}{j\,(X_e + X_q)} = \frac{j\, E_q - V_b\,(\sin\delta + j\cos\delta)}{j\,(X_e + X_q)}$$

$$= \frac{\psi_{fd} + (X_q - X_d')\, I_d - V_b \cos\delta + j\, V_b \sin\delta}{X_e + X_q} \qquad \text{(A-3.1)}$$

From real and imaginary parts of (A-3.1), equations as follows are conducted.

$$I_d = \frac{\psi_{fd} - V_b \cos\delta}{X_e + X_d'} \qquad \text{(A-3.2)}$$

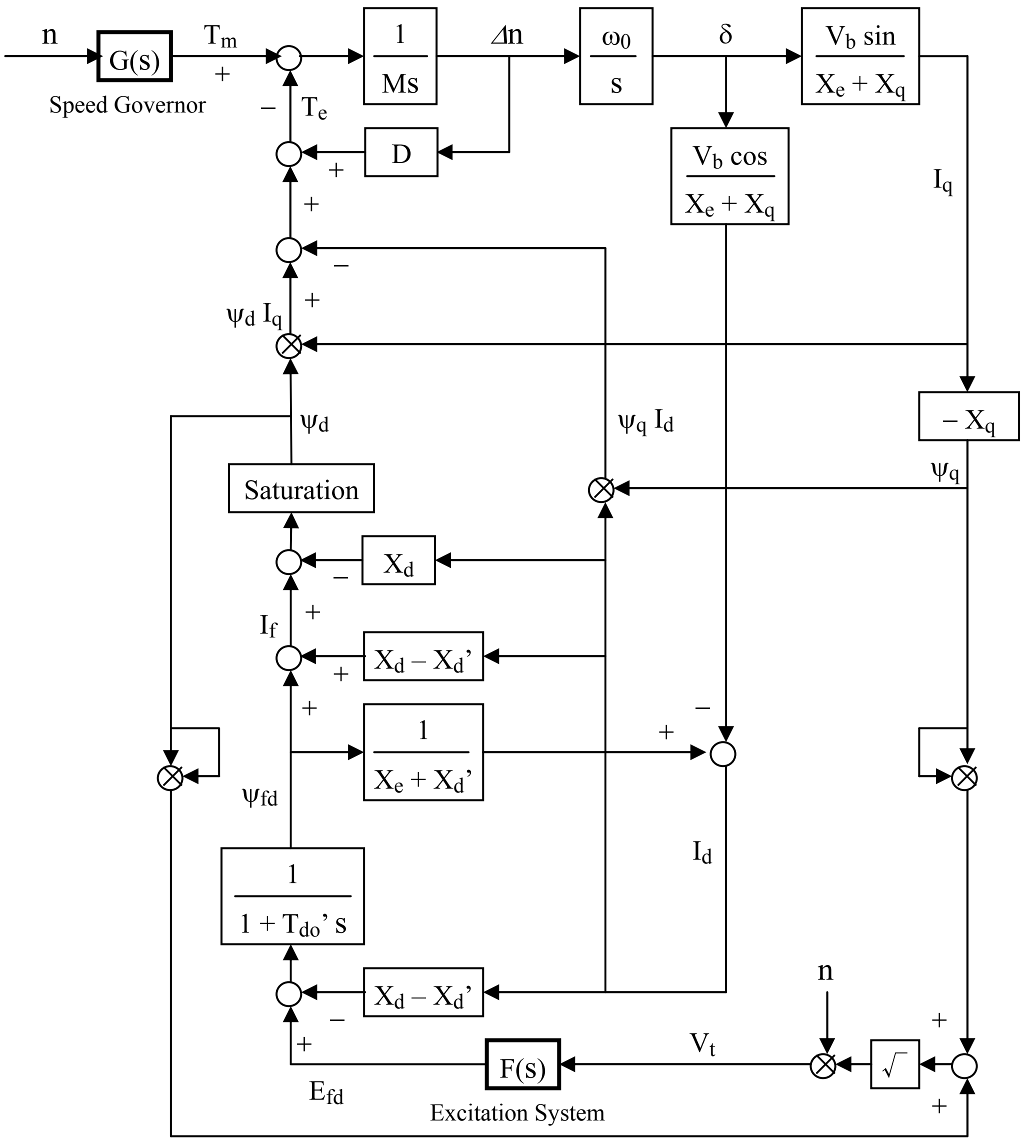
$$I_q = \frac{V_b \sin\delta}{X_e + X_q} \qquad \text{(A-3.3)}$$

Erasing I_f from (A-2.21) and (A-2.24), equation as follows is obtained.

$$\psi_{fd} = \frac{E_{fd} - (X_d - X_d')\, I_d}{1 + T_{do}'\, s} \tag{A-3.4}$$

(A-2.21) is also be written as follows.

$$I_f = \psi_{fd} + (Xd - Xd')\, Id \tag{A-3.5}$$



A-Fig. 3.3 Block diagram of 1 machine and infinite bus model

Relations above can be expresses as block diagram shown in A-Fig. 3.3. Since magnetic saturation appears only in direct axis in case of salient pole machine, saturation element should locate as the figure. Then, direct axis synchronous reactance must be unsaturated value. The block diagram preserves nonlinearity of synchronous machine, and was adopted in Training Hydropower Simulator in 1996.

[Linearization for small disturbance] Oscillatory stability studies system response by small disturbance. Small signal response of A-Fig. 3.3 can be conducted by making relations between differentials of variables. The procedures are as follows. Variables within { } are so small that they are usually ignored. Variables indicating operating point have suffix 0, and they are constants. Infinite bus voltage V_b is, of course, a constant.

Armature circuit is expressed as follows.

$$\varDelta V_d = \{ s\,\varDelta\psi_d \} - \varDelta\psi_q - \{ \psi_q\, s\,\varDelta\delta \} \qquad \text{(A-3.6)}$$

$$\varDelta V_q = \{ s\,\varDelta\psi_q \} - \varDelta\psi_d - \{ \psi_d\, s\,\varDelta\delta \} \qquad \text{(A-3.7)}$$

$$\varDelta\psi_d = \varDelta I_f - X_d\,\varDelta I_d \qquad \text{(A-3.8)}$$

$$\varDelta\psi_q = - X_q\,\varDelta I_q \qquad \text{(A-3.9)}$$

From vector diagram, relations as follows are conducted.

$$\mathbf{V}_t = V_d + jV_q = \mathbf{V}_b + j\,X_e\,I \;= V_b \sin\delta + j\cos\delta + j\,X_e\,(I_d + j\,I_q)$$

$$\therefore \quad V_d = V_b \sin\delta - X_e\,I_q \qquad \text{(A-3.10)}$$

$$V_q = V_b \cos\delta + X_e\,I_d \qquad \text{(A-3.11)}$$

Their differentials are expressed as follows.

$$\varDelta V_d = V_b \cos\delta_0\,\varDelta\delta - X_e\,\varDelta I_q \qquad \text{(A-3.12)}$$

$$\varDelta V_q = - V_b \sin\delta_0\,\varDelta\delta + X_e\,\varDelta I_d \qquad \text{(A-3.13)}$$

Terminal voltage is expressed as follows.

$$V_t^2 = V_d^2 + V_q^2$$

Its differential is expressed as follows.

$$2V_{t0}\,\varDelta V_t = 2V_{d0}\,\varDelta V_d + 2V_{q0}\,\varDelta V_q$$

$$\therefore \quad \varDelta Vt = \frac{V_{d0}}{V_{t0}}\,\varDelta V_d + \frac{V_{q0}}{V_{t0}}\,\varDelta V_q \qquad \text{(A-3.14)}$$

Field circuit is expressed as follows.

$$\varDelta\psi_{fd} = \varDelta I_f - (X_d - X_d')\,\varDelta I_d \qquad \text{(A-3.15)}$$

$$\varDelta E_{fd} = \varDelta I_f + T_{do}'\,s\,\varDelta\psi_{fd} \qquad \text{(A-3.16)}$$

As swing equation, equation as follows are conducted from differentials of (A-2.15) and (付 2.16).

$$\frac{M s^2 + D s}{\omega_0}\,\varDelta\delta = \varDelta T_m + \psi_{q0}\,\varDelta I_d + I_{d0}\,\varDelta\psi_q - \psi_{d0}\,\varDelta I_q - I_{q0}\,\varDelta\psi_d \qquad \text{(A-3.16)}$$

Variables except ΔV_t , $\Delta\delta$, $\Delta\psi_{fd}$, ΔT_m , ΔE_{fd} are erased. At first using (A-3.7), (A-3.8), (A-3.13), ΔV_q and $\Delta\psi_d$ are erased. Then,

$$\Delta I_f - X_d \Delta I_d = - V_b \sin \delta_0 \Delta\delta + X_e \Delta I_d$$

Substituting (A-3.5) to equation above, ΔI_f is erased and ΔI_d is expressed as follows.

$$\Delta I_d = \frac{V_b \sin \delta_0}{X_d' + X_e} \Delta\delta + \frac{1}{X_d' + X_e} \Delta\psi_{fd} \qquad \text{(A-3.18)}$$

From (A-3.6), (A-3.9), and (A-3.12) ΔV_d and $\Delta\psi_q$ are erased as follows.

$$X_q \Delta I_q = V_b \cos \delta_0 \Delta\delta - X_e \Delta I_q$$

$$\Delta I_q = \frac{V_b \cos \delta_0}{X_q + X_e} \qquad \text{(A-3.19)}$$

Substituting (A-3.15) and (A-3.18) to (A-3.7), ΔI_f and ΔI_d are erased and $\Delta\psi_d$ is expressed as follows.

$$\Delta\psi_d = \frac{- X_d' V_b \sin\delta_0}{X_d' + X_e} \Delta\delta + \frac{X_e}{X_d' + X_e} \Delta\psi_{fd} \qquad \text{(A-3.20)}$$

Substituting (A-3.19) to (A-3.9), ΔI_q is erased and $\Delta\psi_q$ is expressed as follows.

$$\Delta\psi_q = \frac{- X_q V_b \cos \delta_0}{X_q + X_e} \Delta\delta \qquad \text{(A-3.21)}$$

(A-2.4') and (A-2.5') are expressed as follows at operating point.

$$\psi_{d0} = V_{q0} \qquad \text{(A-3.22)}$$

$$\psi_{q0} = - V_{d0} \qquad \text{(A-3.23)}$$

(A-3.10) and (A-3.11) are expressed at operating point as follows.

$$I_{d0} = \frac{V_{q0} - V_b \cos \delta_0}{X_e} \qquad \text{(A-3.24)}$$

$$I_{q0} = \frac{- V_{d0} + V_b \sin \delta_0}{X_e} \qquad \text{(A-3.25)}$$

Substituting (付 3.18) - (付 3.25) to (付 3.17), equation as follows is obtained.

$$\frac{M s^2 + D s}{\omega_0} \Delta\delta = \Delta T_m - K_1 \Delta\delta + K_2 \Delta\psi_{fd} \qquad \text{(A-3.26)}$$

Here,

$$K_1 = \frac{-V_{d0}\,V_b \sin\delta_0}{X^{d'} + X_e} + \frac{-X_q\,V_{q0}\,V_b \cos\delta_0}{X_e\,(X_q + X_e)} + \frac{X_q\,V_b^2 \cos^2\delta_0}{X_e\,(X_d' + X_e)}$$

$$+ \frac{-V_{q0}\,V_b \cos\delta_0}{X_q + X_e} + \frac{-X_d'\,V_{q0}\,V_b \cos\delta_0}{X_e\,(X_q + X_e)} + \frac{X_d'\,V_b^2 \sin^2\delta_0}{X_e\,(X_d' + X_e)}$$

$$= \frac{V_b \cos\delta_0\,\{X_q\,V_b \cos\delta_0 - (X_q + X_e)\,V_{q0}\}}{X_e\,(X_q + X_e)} + \frac{V_b \sin\delta_0\,\{X_d'\,V_b \sin\delta_0 - (X_d' + X_e)\,V_{d0}\,\}}{X_e\,(X_d' + X_e)}$$

Equations as follows conducted from (A-3.10) and (A-3.11)

$$V_b \sin\delta_0 = V_{d0} + X_e\,I_{q0}$$

$$V_b \cos\delta_0 = V_{q0} - X_e\,I_{d0}$$

and relations

$$E_{q0} = V_{q0} + X_q\,I_{d0}$$

$$I_{q0} = V_{d0}\,/X_q$$

are adopted and K_1 is expressed as follows.

$$K_1 = \frac{E_{q0}\,V_b \cos\delta_0}{X_q + X_e} + \frac{X_q - X_d'}{X_q} * \frac{V_{d0}\,V_b \sin\delta_0}{X_d' + X_e} \qquad \text{(A-3.27)}$$

K_2 is expressed as follows.

$$K_2 = \frac{V_b \sin\delta_0}{X_d' + X_e} \qquad \text{(A-3.28)}$$

Erasing ΔI_f from equations (A-3.15) and (A-3.16), equation as follows is obtained.

$$(1 + T_{do}'\,s)\,\Delta\psi_{fd} = \Delta E_{fd} - (X_d - X_d')\,\Delta Id$$

Substituting (A-3.18) to equation above, ΔI_d is erased and equation as follows is conducted.

$$\Delta\psi_{fd} = \frac{K_3}{1 + T_d'\,s}\,\Delta E_{fd} - \frac{K_4}{1 + T_d'\,s}\,\Delta\delta \qquad \text{(A-3.29)}$$

Here,

$$T_d' = \frac{X_d' + X_e}{X_d + X_e}\,T_{do}' \qquad \text{(A-3.30)}$$

$$K_3 = \frac{X_d' + X_e}{X_d + X_e} \qquad \text{(A-3.31)}$$

$$K_4 = \frac{X_d - X_d'}{X_d + X_e}\,V_b \sin\delta_0 \qquad \text{(A-3.32)}$$

At last substituting (A-3.6) and (A-3.7) to (A-3.14), equation as follows is obtained.

$$\Delta V_t = -\frac{V_{d0}}{V_{t0}} \Delta\psi q + \frac{V_{d0}}{V_{t0}} \Delta\psi d \qquad \text{(A-3.33)}$$

Substituting (A-3.20) and (A-3.21) to equation above, $\Delta\psi_q$ and $\Delta\psi_d$ are erased and equation as follows is obtained.

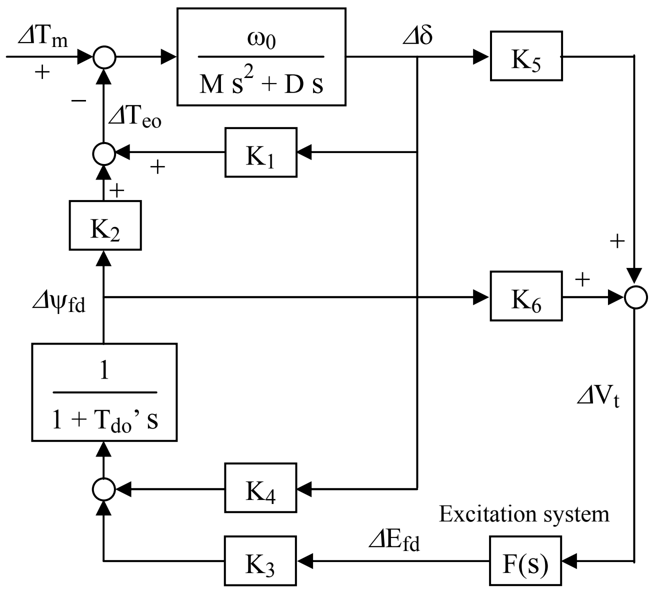
$$\Delta V_t = K_5 \,\Delta\delta + K_6 \,\Delta\psi_{fd}$$

Here,

$$K_5 = \frac{V_{d0}}{V_{t0}} * \frac{X_q V_b \cos\delta_0}{X_q + X_e} - \frac{V_{q0}}{V_{t0}} * \frac{X_d' V_b \sin\delta_0}{X_d + X_e} \qquad \text{(A-3.34)}$$

$$K_6 = \frac{V_{q0}}{V_{t0}} * \frac{X_e}{X_d' + X_e} \qquad \text{(A-3.35)}$$

Thus, all relations between ΔV_t , $\Delta\delta$, $\Delta\psi_{fd}$, ΔT_m , ΔE_{fd} are obtained. Those relations are summarized as block diagram (A-Fig. 3.4).



A-Fig. 3.4　Linearized block diagram

[Damping Torque Coefficient]　In A-Fig. 3.4, electrical output change excluding damping D is expressed as follows.

$$\Delta T_{eo} = K_1 \,\Delta\delta + K_2 \,\Delta\psi_{fd} = (K_S + jK_D)\,\Delta\delta$$

Here, K_S and K_D are functions of power swing angular frequency ω_s , and called as synchronizing torque coefficient and damping torque coefficient respectively.

Transfer function from mechanical input change ΔT_m to rotor angle change $\Delta\delta$ in A-Fig. 3.4 is expressed as follows.

$$\Delta\delta = \frac{1}{\{K_S - M\,\omega_s/\omega_0\} + j\,\{K_D + D\,\omega_s/\omega_0\}}\,\Delta T_m \qquad \text{(A-3.36)}$$

In denominator of (A-3.36), real part means synchronizing torque and imaginary part means damping torque. Their sign mean as follows.

If real part is positive, generator does not step out Imaginary part is positive, oscillation decay.

If real part is negative, generator does step out Imaginary part is negative, oscillation grow.

Damping torque coefficient K_D is calculated as follows.

$$\Delta\psi_{fd} = \frac{K_3\,F(s)\,\{K_5\,\Delta\delta + K_6\,\Delta\psi_{fd}\} - K_4\,\Delta\delta}{1 + T_d'\,s}$$

$$\therefore \quad \Delta\psi_{fd} = \frac{K_3\,F(s)\,K_5 - K_4}{1 + T_d'\,\tilde{s}\;K_3\,F(s)\,K_6}\,\Delta\delta \qquad \text{(A-3.37)}$$

Therefore, synchronizing and damping torques are expressed as follows.

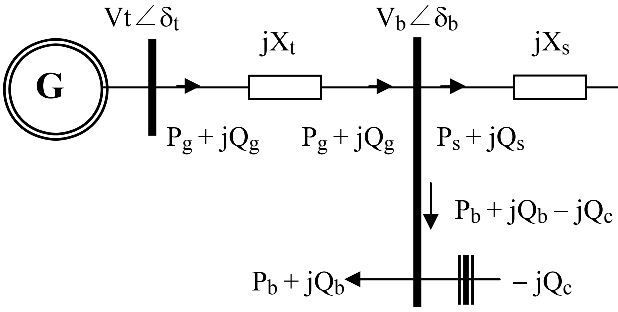
$$K_S + j\,K_D = K_1 + \frac{K_2\,\Delta\psi_{fd}}{\Delta\delta} = K_1 + \frac{K_2\,\{K_3\,F(s)\,K_5\,K_4}{1 + T_d'\,s\,K_3\,F(s)\,K_6} \qquad \text{(A-3.38)}$$

Here, considering that gain of excitation system is $-F(s) = F_r + jF_i$ and ts = jω, damping torque is expressed as follows.

$$K_D = K_2\,\frac{K_3\,K_5\,Fr\,(1 + K_3\,K_6\,F_r) + (K_3\,K_5\,F_r + K_4)\,(\omega T_d' - K_3\,F_i)}{(1 + K_3\,K_6\,F_r) + (\omega T_d' - K_3\,F_i)} \qquad \text{(A-3.39)}$$

A-4 1 Machine, 1 Load, and Infinite Bus Model

[Block Diagram of Small Variations] Power system that has a load between a generator and infinite bus as shown in A-Fig. 4.1 is studied. Most subsystems make the form when aggregated. Therefore, so as to the subsystem studied includes loads, 1 machine, 1 load, and infinite bus model is the simplest description. Influence by load branch impedance can be included into load's voltage sensitivity seen from load’s branching point. Although the model is added only 1 load from 1 machine and infinite bus model, analysis becomes highly complex.



A-Fig. 4.1 1 machine, 1 load, and infinite bus model

Power flow conditions of A-Fig. 4.1 are listed up as follows.

$$P_g = \frac{V_t V_b \sin(\delta_t - \delta_b)}{X_t}, \quad Q_t = \frac{V_t^2 - V_t V_b \cos(\delta_t - \delta_b)}{X_t}, \quad Q_g = \frac{V_t V_b \cos(\delta_t - \delta_b) - V_b^2}{X_t}$$

$$P_s = \frac{V_b V_s \sin \delta_b}{X_s}, \quad Q_s = \frac{V_b^2 - V_b V_s \cos \delta_b}{X_s}$$

Small variations around operational point of these variables are expressed as follows.

$$\Delta P_g = P_g \frac{\Delta V_t}{V_t} + P_g \frac{\Delta V_b}{V_b} + \left(Q_g + \frac{V_b^2}{X_t} \right) (\Delta\delta_t - \Delta\delta_b) \qquad \text{(A-4.1)}$$

$$\Delta Q_t = \left(Q_t + \frac{V_t^2}{X_t} \right) \frac{\Delta V_t}{V_t} + \left(Q_t - \frac{V_t^2}{X_t} \right) \frac{\Delta V_b}{V_b} + Pg\, (\Delta\delta_t - \Delta\delta_b) \qquad \text{(A-4.2)}$$

$$\Delta Q_g = \left(Q_g + \frac{V_b^2}{X_t} \right) \frac{\Delta V_t}{V_t} + \left(Q_g - \frac{V_b^2}{X_t} \right) \frac{\Delta V_b}{V_b} - P_g (\Delta\delta_t - \Delta\delta_b) \qquad \text{(A-4.3)}$$

$$\Delta P_s = P_s \frac{\Delta V_b}{V_b} - \left(Q_s - \frac{V_b^2}{X_s} \right) \Delta\delta_b \qquad \text{(A-4.4)}$$

$$\Delta Q_s = \left(Q_s + \frac{V_b^2}{X_s} \right) \frac{\Delta V_b}{V_b} + Ps\, \Delta\delta_b \qquad \text{(A-4.5)}$$

Small variations of the load are expresses as follows by assuming voltage sensitivities of load's active, reactive load and capacitor as α , β , γ respectively.

$$\Delta P_b = \alpha\, P_b \frac{\Delta V_b}{V_b} \quad , \quad \Delta Q_b = \beta\, Q_b \frac{\Delta V_b}{V_b} \quad , \quad \Delta Q_c = \gamma\, Q_c \frac{\Delta V_b}{V_b} \qquad \text{(A-4.6)}$$

At load's branching point, load balance conditions of small variations are expressed as follows.

$$\Delta P_g = \Delta P_b + \Delta P_s \quad , \quad \Delta Q_g = \Delta Q_b + \Delta Q_c + \Delta Q_s$$

Thus, equations as follows are conducted.

$$\begin{pmatrix} \frac{V_b^2}{X_t} + \frac{V_b^2}{X_s} + Q_g - Q_s & P_s - P_g + \alpha\, P_b \\ P_s - P_g & \frac{V_b^2}{X_t} + \frac{V_b^2}{X_s} + Q_g - Q_s + \beta\, Q_b - \gamma\, Q_c \end{pmatrix} \begin{pmatrix} \Delta\delta_b \\ \frac{\Delta V_b}{V_b} \end{pmatrix} = \begin{pmatrix} \frac{V_b^2}{X_t} + Q_g & P_g \\ -Pg & \frac{V_b^2}{X_t} + Q_g \end{pmatrix} \begin{pmatrix} \Delta\delta_t \\ \frac{\Delta V_t}{V_t} \end{pmatrix} \qquad \text{(A-4.7)}$$

Using the equations, 2 small variations out of the all 4 can be erased. Since we want to know relations between the machine and the infinite bus, variations that should be erased are those of load's branching point. Therefore, making calculation on lines, matrix in left side is translated to unit matrix, and the equations are expressed as follows, which is used to erase $\Delta\delta_b$ and ΔV_b .

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\delta_b \\ \frac{\Delta V_b}{V_b} \end{pmatrix} = \begin{pmatrix} A_{13} & A_{14} \\ A_{23} & A_{24} \end{pmatrix} \begin{pmatrix} \Delta\delta_t \\ \frac{\Delta V_t}{V_t} \end{pmatrix} \qquad \text{(A-4.8)}$$

Substituting (A-4.8) to (A-4.1) and (A-4.2), $\Delta\delta_b$ and ΔV_b are erased and equations as follows is obtained.

$$\Delta P_g = \{P_g A_{23} + (\frac{V_t^2}{X_t} - Q_t)(1 - A_{13})\}\, \Delta\delta_t + \{P_g (1 + A_{24}) - (\frac{V_t^2}{X_t} - Q_t) A_{14}\} \frac{\Delta V_t}{V_t}$$

$$= P_{g1}\, \Delta\delta_t + P_{g2} \frac{\Delta V_t}{V_t} \qquad \text{(A-4.9)}$$

$$\Delta Q_t = \{(Q_g - \frac{V_t^2}{X_t}) A_{23} + P_g (1 - A_{13})\}\, \Delta\delta_t + \{Q_t (1 + A_{24}) + (1 - A_{24}) - P_g A_{14}\} \frac{\Delta V_t}{V_t}$$

$$= Q_{t1}\, \Delta\delta_t + Q_{t2} \frac{\Delta V_t}{V_t} \qquad \text{(A-4.10)}$$

The generator is expressed as follows.

$$E_q = \sqrt{\ } \{(V_t + X_q \frac{Q_t}{V_t})^2 + (X_q \frac{Q_t}{V_t})^2\}$$

$$\sin(\delta - \delta_t) = \frac{X_q P_g}{E_q V_t}$$

$$P_g + j\,Q_t = (V_d I_d + V_q I_q) + j\,(V_q I_d + V_d I_q)$$

$$I_d = \frac{P_g \sin(\delta - \delta_t)}{V_t} + \frac{Q_t \cos(\delta - \delta_t)}{V_t}$$

$$I_q = \frac{P_g \cos(\delta - \delta_t)}{Vt} - \frac{Q_t \sin(\delta - \delta_t)}{V_t}$$

$$V_d = X_q I_q$$

$$V_q = \sqrt{(V_t^2 - V_d^2)} = \psi_{fd} - X_d' I_d$$

Using coefficients P_{g1} , P_{g2} , Q_{t1} , Q_{t2} in (A-4.9) and (A-4.10), small variation of I_d and I_q are expressed as follows.

$$\begin{aligned}\Delta I_d = {} & \frac{P_{g1} \sin(\delta - \delta_t) + Q_{t1} \cos(\delta - \delta_t) - P_g \cos(\delta - \delta_t) + Q_t \sin(\delta - \delta_t)}{V_t} \Delta\delta_t \\ & + \frac{P_{g2} \sin(\delta - \delta_t) + Q_{t2} \cos(\delta - \delta_t) - P_g \sin(\delta - \delta_t) - Q_t \cos(\delta - \delta_t)}{V_t} \frac{\Delta V_t}{V_t} \\ & + \frac{P_g \cos(\delta - \delta_t) - Q_t \sin(\delta - \delta_t)}{V_t} \Delta\delta \\ = {} & I_{d1}\,\Delta\delta_t + I_{d2} \frac{\Delta V_t}{V_t} + I_{d3}\,\Delta\delta \end{aligned} \quad \text{(A-4.11)}$$

$$\begin{aligned}\Delta I_q = {} & \frac{P_{g1} \cos(\delta - \delta_t) - Q_{t1} \sin(\delta - \delta_t) + P_g \sin(\delta - \delta_t) + Q_t \cos(\delta - \delta_t)}{V_t} \Delta\delta_t \\ & + \frac{P_{g2} \cos(\delta - \delta_t) - Q_{t2} \sin(\delta - \delta_t) - P_g \cos(\delta - \delta_t) + Q_t \sin(\delta - \delta_t)}{V_t} \frac{\Delta V_t}{V_t} \\ & + \frac{P_g \sin(\delta - \delta_t) - Q_t \cos(\delta - \delta_t)}{V_t} \Delta\delta \\ = {} & I_{q1}\,\Delta\delta_t + I_{q2} \frac{\Delta V_t}{V_t} + I_{q3}\,\Delta\delta \end{aligned} \quad \text{(A-4.12)}$$

Small variation of V_d and V_q are expressed as follows. 。

$$\Delta V_d = X_q\,\Delta I_q \quad \text{(A-4.13)}$$

$$\Delta V_q = \Delta\psi_{fd} - X_d'\,\Delta I_d \quad \text{(A-4.14)}$$

Small variation of P_g and Q_t are calculated as follows.

$$\Delta P_g = V_d\,\Delta I_d + V_q\,\Delta I_q + I_d\,\Delta V_d + I_q\,\Delta V_q \quad \text{(A-4.15)}$$

$$\Delta Q_t = V_q\,\Delta I_d - V_d\,\Delta I_q - I_q\,\Delta V_d + I_d\,\Delta V_q \qquad \text{(A-4.16)}$$

Substituting (A-4.9) - (A-4.14) to (A-4.15) and (A-4.16), relations as follows are obtained.

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{11} & B_{12} \end{pmatrix} \begin{pmatrix} \Delta\delta_t \\ \dfrac{\Delta V_t}{V_t} \end{pmatrix} = \begin{pmatrix} B_{13} & B_{14} \\ B_{23} & B_{24} \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta\psi_{fd} \end{pmatrix} \qquad \text{(A-4.17)}$$

Elements of the matrix above are expressed as follows.

$$B_{11} = P_{g1} - V_d\,I_{d1} - V_q\,I_{q1} - I_d\,X_q\,I_{q1} + I_q\,X_d'\,I_{d1}$$

$$B_{12} = P_{g2} - V_d\,I_{d2} - V_q\,I_{q2} - I_d\,X_q\,I_{q2} + I_q\,X_d'\,I_{d2}$$

$$B_{13} = \qquad V_d\,I_{d3} + V_q\,I_{q3} + I_d\,X_q\,I_{q3} - I_q\,X_d'\,I_{d3}$$

$$B_{14} = \qquad I_q$$

$$B_{21} = Q_{t1} - V_q\,I_{d1} + V_d\,I_{q1} + I_d\,X_d'\,I_{d1} + I_q\,X_q\,I_{q1}$$

$$B_{22} = Q_{t2} - V_q\,I_{d2} + V_d\,I_{q2} + I_d\,X_d'\,I_{d2} + I_q\,X_q\,I_{q2}$$

$$B_{23} = \qquad V_q\,I_{d3} - V_d\,I_{q3} - I_d\,X_d'\,I_{d3} - I_q\,X_q\,I_{q3}$$

$$B_{24} = \qquad I_d$$

By making calculation only on lines, matrix on the left side (A-4.17) can also be transformed to unit matrix as follows. Although same characters as (A-4.17) are used, of course, those values are different.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\delta_t \\ \dfrac{\Delta V_t}{V_t} \end{pmatrix} = \begin{pmatrix} B_{13} & B_{14} \\ B_{23} & B_{24} \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta\psi_{fd} \end{pmatrix} \qquad \text{(A-4.18)}$$

Substituting (A-4.18) to (付 4.9), equation as follows is obtained.

$$\Delta P_g = P_{g1}\left(B_{13}\,\Delta\delta + B_{14}\,\frac{\Delta V_t}{V_t} \right) + P_{g2}\left(B_{23}\,\Delta\delta + B_{24}\,\frac{\Delta V_t}{V_t} \right)$$

$$= K_1\,\Delta\delta + K_2\,\Delta V_t \qquad \text{(A-4.19)}$$

Here,

$$K_1 = P_{g1}\,B_{13} + P_{g2}\,B_{23} \qquad \text{(A-4.20)}$$

$$K_2 = (P_{g1}\,B_{14} + P_{g2}\,B_{24}) / V_t \qquad \text{(A-4.21)}$$

Field flux is most fundamentally expressed as follows.

$$(1+ T_{do}'\, s)\ \Delta\psi_{fd} = \Delta E_{fd} - (X_d - X_d')\ \Delta I_d$$

$$= \Delta E_{fd} - (X_d - X_d')\ (I_{d1}\ \Delta\delta_t + I_{d2}\ \frac{\Delta V_t}{V_t} + I_{d3}\ \Delta\delta)$$

$$= \Delta E_{fd} - (X_d - X_d')\ \{ I_{d1}\ (B_{13}\ \Delta\delta + B_{14}\ \Delta\psi_{fd}) + I_{d2}\ (B_{23}\ \Delta\delta + B_{24}\ \Delta\psi_{fd}) + I_{d3}\ \Delta\delta\}$$

$$\therefore \quad T_{do}'\, s\ \Delta\psi_{fd} = \Delta E_{fd} - K_3\ \Delta\psi_{fd} - K_4\ \Delta\delta \qquad \text{(A-4.22)}$$

Here,

$$K_3 = 1 + (X_d - X_d')\ (I_{d1}\ B_{14} + I_{d2}\ B_{24}) \qquad \text{(A-4.23)}$$

$$K_4 = (X_d - X_d')\ (I_{d1}\ B_{13} + I_{d2}\ B_{23} + I_{d3}) \qquad \text{(A-4.24)}$$

From (A-4.18), relation as follows is directly obtained.

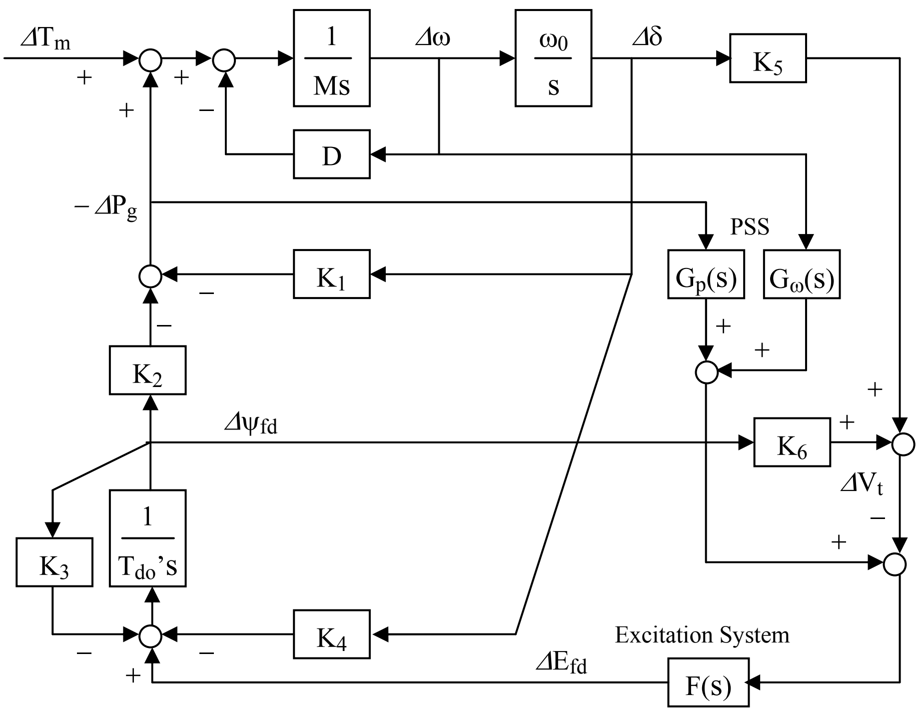
$$\Delta V_t = K_5\ \Delta\delta + K_6\ \Delta\psi_{fd} \qquad \text{(付 4.25)}$$

Here,

$$K_5 = V_t\ B_{23} \qquad \text{(付 4.26)}$$

$$K_6 = V_t\ B_{24} \qquad \text{(付 4.27)}$$

Relations (A-4.19) to (A-4.27) are synthesized into block diagram as A-Fig. 4.2, which has the same form of A-Fig. 3.4, but K parameters turn from constants defined by operating points to functions of load's voltage sensitivity. Therefore, the model is an expansion of de Mello's model. As to the slight different of expression on field, they are equivalent but the figure is more radical.



A-Fg. 4.2 Linearized block diagram of 1 machine, 1 load and infinite bus

Relations around synchronizing and damping torque coefficients K_s and K_d are obtained as follows.

$$(K_3 + T_{do}'\, s)\, \Delta\psi_{fd} = -\, F(s)\, (\, K_5\, \Delta\delta + K_6\, \Delta\psi_{fd} + G_{pss}(s)\, \Delta P_g) - K_4\, \Delta\delta$$

$$\therefore \quad \Delta\psi_{fd} = \frac{\{-\, F(s)\, K_5 - K_4\}\, \Delta\delta - F(s)\, G_{pss}(s)\, \Delta P_g}{K_3 + T_{do}'\, s + F(s)\, K_6} \qquad \text{(A-4.28)}$$

$$\Delta P_g = K_1\, \Delta\delta + K_2\, \Delta\psi_{fd}$$

$$= (K_1 + \frac{K_2\, F(s)\, K_5 - K_2\, K_4}{K_3 + T_{do}'\, s + F(s)\, K_6}\,)\, \Delta\delta + \frac{-\, K_2\, F(s)\, G_{pss}(s)}{K_3 + T_{do}'\, s + F(s)\, K_6}\, \Delta P_g$$

$$\therefore \quad \Delta P_g = \frac{K_1 + \dfrac{K_2\, F(s)\, K_5 - K_2\, K_4}{K_3 + T_{do}'\, s + F(s)\, K_6}}{1 + \dfrac{K_2\, F(s)\, G_{pss}(s)}{K_3 + T_{do}'\, s + F(s)\, K_6}}\, \Delta\delta$$

$$= \frac{K_1\, \{K_3 + T_{do}'\, s + F(s)\, K_6\} - K_2\, F(s)\, K_5 - K_2\, K_4}{K_3 + T_{do}'\, s + F(s)\, K_6 + K_2\, F(s)\, G_{pss}(s)}$$

$$= (K_s + j\, K_d\,)\, \Delta\delta \qquad \text{(A-4.29)}$$

Thus, synchronizing and damping torque coefficients K_s and K_d are obtained. Here, $G_{pss}(s)$ means PSS gain translated to ΔP type. Total transfer function from ΔT_m to $\Delta\delta$ is conducted as follows.

$$\frac{\Delta\delta}{\Delta T_m} = \frac{\dfrac{\omega_0}{-\,\omega^2 M + j\, \omega\, D}}{1 + \dfrac{\omega_0}{-\,\omega^2 M + j\, \omega\, D}\, (K_s + j\, K_d)}$$

$$= \frac{1}{(\, K_s - \dfrac{\omega^2 M}{\omega_0}\,) + j\, (\, K_d + \dfrac{\omega D}{\omega_0}\,)} \qquad \text{(A-4.30)}$$

Power swing angular frequency of the subsystem ω_s is calculated by making denominator's real part of (A-4.30) zero as follows.

$$\omega_0\, K_s - \omega_s^2\, M = 0$$

$$\therefore \quad \omega_s = \sqrt{(\, \omega_0\, K_s/M\,)} \qquad \text{(A-4.31)}$$

At any power swing angular frequency ω, damping torque coefficient of the subsystem K_d' is conducted , by adding damping of the generator itself: $K_{d0} = \omega D/\omega_0$, as follows.

$$K_d'(\omega) = K_d(\omega) + \omega D/\omega_0 \qquad \text{(A-4.32)}$$

Postscript

The author has been astonished, because results by extended classic analysis methods based on simply aggregated power system model has given almost same results by modern simulation tool on detailed power system model. Predecessors who could not use modern simulation tools were forced to develop and use classic analysis methods, which are proved quite adequate if correctly used. Those classic and extended classic analysis methods can give engineers insights, which modern simulation tools cannot. Further saying, only those who use classic analyses methods have qualification for performing simulation, because simulation without physical insights always has risk for mistaking and overlooking the mistake. However, in spite of its importance, classic theories are going to lose their initiators. *Buddhism* had predicted such a condition of itself as *Mappo*, which means decline of the doctrines, dieing out of initiators, and only remaining of the *Scriptures*. *Buddhism* had also predicted that *Mappo* would go worse to *Meppo*, which means complete ruin of the doctrines and the *Scriptures*. The author has published the book as a *Scripture*, and hopes that it will survive as long as possible in *Mappo* era of electric power system engineering.

By the way, why such an important technology is going to ruin? One reason is thought that accomplishment of simulation tools has reduced importance of classic theories. Since everybody will believe simulation result, engineers do not necessarily have to know theories and physical meanings, only if simulation runs to the end. By *Confucian* criticism, simulation technology has ruined engineers who developed it. Thus, simulation is in all its glory today. But let us think a little while. Simulation tools does not run and give answer if users do not put in numerical data. Users are not always expert engineers, but only masters of TV game named simulation and inhabitants of virtual reality of simulation. It is quite possible that users cannot examine the adequacy of model and data to be put into the tools. In such a condition, who can rely on the answers given by such simulations? Those who trust upon them might be sleeping on volcano mouth.

Electric power system is metamorphosing. Today’s adequate model and data cannot always be adequate in tomorrow. The author think that now is the metamorphosing period. Two main incidents are progressing. One is retirement of aged thermal generation. Another is penetration of distributed generation. Both of the two will reduce load’s voltage stability, and as the results, reduce transient, dynamic, and frequency stability of interconnection. Traditional model and method have been proved not to tell the truth but to overlook and mislead. As the countermeasure, the author has introduced new model and method. The results were astonishing. Besides, although everybody understands that high penetration of RE whose output fluctuates by time will threaten voltage and frequency regulation in power system, quantitatively analyses seems to stagnate, therefore, the author by himself clarified. The contents were published as paper and introduced in the book.

It is “Science of Philosophy” that understand why natural science has achieves such fine success. Science has its unique and brilliant method. To do along with scientific way is very likely to complete account responsibility. However in recent Japan, those who are regarded as scientists sometimes tell what are not scientific. Instant decision as active fault in nuclear plant by “Nuclear Regulation Committee” is typical, and members seem not to complete account responsibility for utilities and self-governing bodies.

But the criticism can be also applied to utilities. Have utilities taken scientific approach and completed account responsibility for an example in problems on RE integration? Among utilities, the author has continued to be scientific. Research fruits have been published as paper with peer review. Peer review is a strong proof that distinguishes those papers from pseudo-science. Announcement by private publishing or technical report without peer review is not validated and sometimes brings useless misleading. Therefore, paper with strong impact must be published as paper with peer review. Although utilities are mainly not sender but receiver of information, they should know the difference between papers (with peer review) and private publishing or technical reports.

Contents dealt in the book are not in high level and not difficult. Ordinary scientists must reach the destinations. Then, why numerous employed scientists cannot reach the destinations before the author? It has been a question for the author long. However recently, it was recognized as possible as follows. That is, employed scientists do not like that now going research theme finishes. More epoch-making the research is, the research becomes higher “destructive creation”, which makes past efforts including outskirts nothing. By “destructive creation”, now going research come to its end and scientists must transfer another theme to live. Considerable effort will be needed. Treatment of employed scientists became worse than ever. Employment with period became ordinary. If the theme finished, extension of employing period may vanish. Therefore, the sense that available time of the theme should be made as long as possible can be understood. But it is national loss. They are non-employed scientists who break such sabotage by employed scientists. Of course, employed scientists oppose to new theories and so on. Contents of the book were also opposed. In Japan, the book may be burned like *Giordano Bruno*. Therefore, this English version is made. In near future, when the country will be in need, these technologies can be reverse-imported. That is *Kurofune* operation. The book will be uploaded in the author’s site.

The book is *Cassandra*’s prediction. Prediction never works if the two conditions are not fulfilled. The first is to realize. The second is to be believed. *Cassandra*’s prediction lacked the second condition. Thus, *Troy* lost out to *Greece* and became a ruin. However, even *Cassandra*’s prediction may be believed outside of *Troy*. And ruin of *Troy* may be discovered by *Schliemann*. Really, the *Cassandra*’s prediction is already spreading in the world as US as the first. In near future, it will become new standard of electric power system engineering. Then, for honor of Japanese engineers, the author intends to leave some evidence that these studies ware performed in the past, although the fruits were executed by fire or buried underground. That is a small hope of the author.

May 2014

An obscure electric engineer

Shintaro Komami